CALCULUS AB

SECTION I, Part A

Time—55 minutes

Number of questions—28

A CALCULATOR MAY NOT BE USED ON THIS PART OF THE EXAM.

Directions: Solve each of the following problems, using the available space for scratch work. After examining the form of the choices, decide which is the best of the choices given and place the letter of your choice in the corresponding box on the student answer sheet. Do not spend too much time on any one problem.

In this exam:

(1) Unless otherwise specified, the domain of a function f is assumed to be the set of all real numbers x for which $f(x)$ is a real number.

(2) The inverse of a trigonometric function f may be indicated using the inverse function notation f^{-1} or with the prefix "arc" (e.g., $\sin^{-1} x = \arcsin x$).

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1. $\int \cos(3x)\,dx =$

(A) $-3\sin(3x) + C$

(B) $-\frac{1}{3}\sin(3x) + C$

(C) $\frac{1}{3}\sin(3x) + C$

(D) $\sin(3x) + C$

(E) $3\sin(3x) + C$

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2. $\lim_{x \to 0} \frac{2x^6 + 6x^3}{4x^5 + 3x^3}$ is

(A) 0 (B) $\frac{1}{2}$ (C) 1 (D) 2 (E) nonexistent

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$$f(x) = \begin{cases} x^2 - 3x + 9 & \text{for } x \le 2 \\ kx + 1 & \text{for } x > 2 \end{cases}$$

3. The function f is defined above. For what value of k, if any, is f continuous at $x = 2$?

(A) 1

(B) 2

(C) 3

(D) 7

(E) No value of k will make f continuous at $x = 2$.

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4. If $f(x) = \cos^3(4x)$, then $f'(x) =$

(A) $3\cos^2(4x)$

(B) $-12\cos^2(4x)\sin(4x)$

(C) $-3\cos^2(4x)\sin(4x)$

(D) $12\cos^2(4x)\sin(4x)$

(E) $-4\sin^3(4x)$

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5. The function f given by $f(x) = 2x^3 - 3x^2 - 12x$ has a relative minimum at $x =$

(A) -1 (B) 0 (C) 2 (D) $\frac{3-\sqrt{105}}{4}$ (E) $\frac{3+\sqrt{105}}{4}$

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6. Let f be the function given by $f(x) = (2x-1)^5(x+1)$. Which of the following is an equation for the line tangent to the graph of f at the point where $x = 1$?

(A) $y = 21x + 2$

(B) $y = 21x - 19$

(C) $y = 11x - 9$

(D) $y = 10x + 2$

(E) $y = 10x - 8$

GO ON TO THE NEXT PAGE.

7. $\int \frac{e^{\sqrt{x}}}{\sqrt{x}}\,dx =$

(A) $2e^{\sqrt{x}} + C$

(B) $\frac{1}{2}e^{\sqrt{x}} + C$

(C) $e^{\sqrt{x}} + C$

(D) $2\sqrt{x}\,e^{\sqrt{x}} + C$

(E) $\frac{1}{2}\frac{e^{\sqrt{x}}}{\sqrt{x}} + C$

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x	0	2	4	6
$f(x)$	4	k	8	12

8. The function f is continuous on the closed interval $[0, 6]$ and has the values given in the table above. The trapezoidal approximation for $\int_0^6 f(x)\,dx$ found with 3 subintervals of equal length is 52. What is the value of k?

(A) 2 (B) 6 (C) 7 (D) 10 (E) 14

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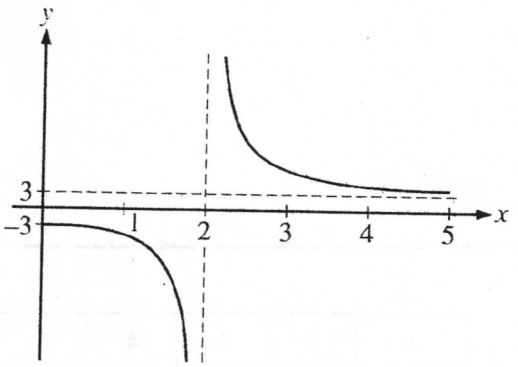
9. A particle moves along the x-axis so that at any time $t > 0$, its velocity is given by $v(t) = 4 - 6t^2$. If the particle is at position $x = 7$ at time $t = 1$, what is the position of the particle at time $t = 2$?

(A) -10 (B) -5 (C) -3 (D) 3 (E) 17



10. The function f is given by $f(x) = \dfrac{ax^2 + 12}{x^2 + b}$. The figure above shows a portion of the graph of f. Which of the following could be the values of the constants a and b ?

(A) $a = -3,\ b = 2$

(B) $a = 2,\ b = -3$

(C) $a = 2,\ b = -2$

(D) $a = 3,\ b = -4$

(E) $a = 3,\ b = 4$

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11. What is the slope of the line tangent to the graph of $y = \dfrac{e^{-x}}{x+1}$ at $x = 1$?

(A) $-\dfrac{1}{e}$ (B) $-\dfrac{3}{4e}$ (C) $-\dfrac{1}{4e}$ (D) $\dfrac{1}{4e}$ (E) $\dfrac{1}{e}$

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12. If $f'(x) = \dfrac{2}{x}$ and $f(\sqrt{e}) = 5$, then $f(e) =$

(A) 2 (B) $\ln 25$ (C) $5 + \dfrac{2}{e} - \dfrac{2}{e^2}$ (D) 6 (E) 25

GO ON TO THE NEXT PAGE.

13. $\int \left(x^3+1\right)^2 dx =$

(A) $\frac{1}{7}x^7 + x + C$

(B) $\frac{1}{7}x^7 + \frac{1}{2}x^4 + x + C$

(C) $6x^2\left(x^3+1\right) + C$

(D) $\frac{1}{3}\left(x^3+1\right)^3 + C$

(E) $\frac{\left(x^3+1\right)^3}{9x^2} + C$

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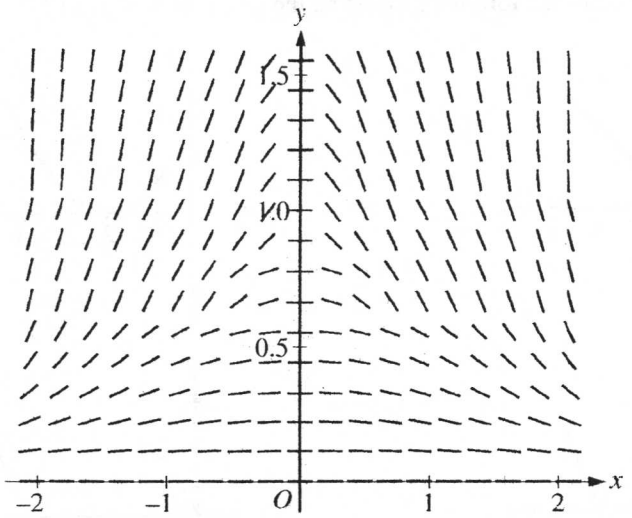
14. $\lim_{h\to 0} \frac{e^{(2+h)} - e^2}{h} =$

(A) 0 (B) 1 (C) $2e$ (D) e^2 (E) $2e^2$



15. The slope field for a certain differential equation is shown above. Which of the following could be a solution to the differential equation with the initial condition $y(0) = 1$?

(A) $y = \cos x$

(B) $y = 1 - x^2$

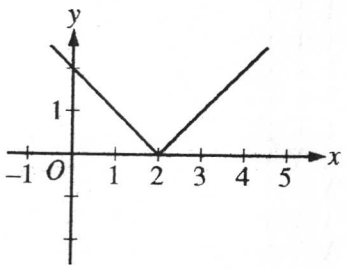
(C) $y = e^x$

(D) $y = \sqrt{1 - x^2}$

(E) $y = \dfrac{1}{1 + x^2}$

16. If $f'(x) = |x - 2|$, which of the following could be the graph of $y = f(x)$?

(A)



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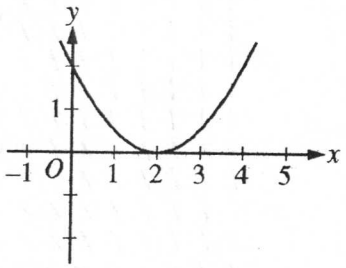

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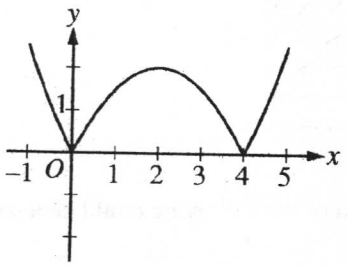
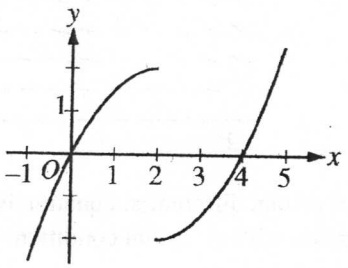
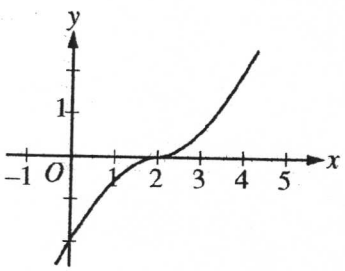


(D)



(E)



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17. What is the area of the region enclosed by the graphs of $f(x) = x - 2x^2$ and $g(x) = -5x$?

(A) $\frac{7}{3}$ (B) $\frac{16}{3}$ (C) $\frac{20}{3}$ (D) 9 (E) 36

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18. For the function f, $f'(x) = 2x + 1$ and $f(1) = 4$. What is the approximation for $f(1.2)$ found by using the line tangent to the graph of f at $x = 1$?

(A) 0.6 (B) 3.4 (C) 4.2 (D) 4.6 (E) 4.64

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19. Let f be the function given by $f(x) = x^3 - 6x^2$. The graph of f is concave up when

(A) $x > 2$

(B) $x < 2$

(C) $0 < x < 4$

(D) $x < 0$ or $x > 4$ only

(E) $x > 6$ only

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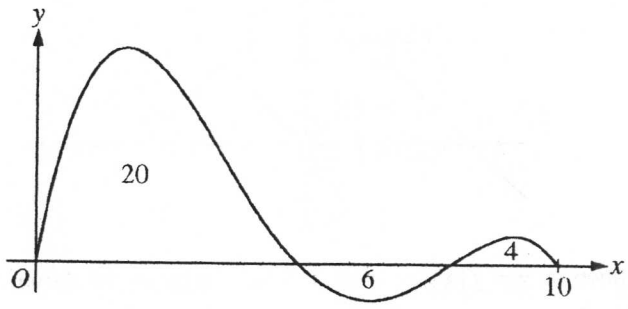
20. If $g(x) = x^2 - 3x + 4$ and $f(x) = g'(x)$, then $\int_1^3 f(x)\,dx =$

(A) $-\frac{14}{3}$ (B) -2 (C) 2 (D) 4 (E) $\frac{14}{3}$



Graph of f'

21. The graph of f', the derivative of the function f, is shown above for $0 \le x \le 10$. The areas of the regions between the graph of f' and the x-axis are 20, 6, and 4, respectively. If $f(0) = 2$, what is the maximum value of f on the closed interval $0 \le x \le 10$?

(A) 16 (B) 20 (C) 22 (D) 30 (E) 32

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22. If $f'(x) = (x-2)(x-3)^2(x-4)^3$, then f has which of the following relative extrema?

I. A relative maximum at $x = 2$

II. A relative minimum at $x = 3$

III. A relative maximum at $x = 4$

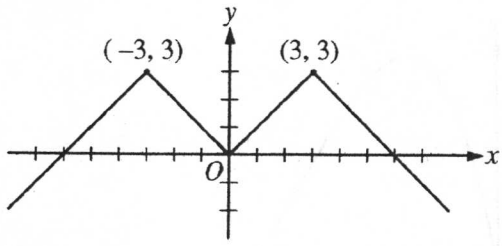
(A) I only

(B) III only

(C) I and III only

(D) II and III only

(E) I, II, and III



23. The graph of the even function $y = f(x)$ consists of 4 line segments, as shown above. Which of the following statements about f is false?

(A) $\lim_{x \to 0} (f(x) - f(0)) = 0$

(B) $\lim_{x \to 0} \frac{f(x) - f(0)}{x} = 0$

(C) $\lim_{x \to 0} \frac{f(x) - f(-x)}{2x} = 0$

(D) $\lim_{x \to 2} \frac{f(x) - f(2)}{x - 2} = 1$

(E) $\lim_{x \to 3} \frac{f(x) - f(3)}{x - 3}$ does not exist.

24. The radius of a circle is increasing. At a certain instant, the rate of increase in the area of the circle is numerically equal to twice the rate of increase in its circumference. What is the radius of the circle at that instant?

(A) $\frac{1}{2}$ (B) 1 (C) $\sqrt{2}$ (D) 2 (E) 4

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25. If $x^2y - 3x = y^3 - 3$, then at the point $(-1, 2)$, $\frac{dy}{dx} =$

(A) $-\frac{7}{11}$ (B) $-\frac{7}{13}$ (C) $-\frac{1}{2}$ (D) $-\frac{3}{14}$ (E) 7

26. For $x > 0$, f is a function such that $f'(x) = \frac{\ln x}{x}$ and $f''(x) = \frac{1 - \ln x}{x^2}$. Which of the following is true?

(A) f is decreasing for $x > 1$, and the graph of f is concave down for $x > e$.

(B) f is decreasing for $x > 1$, and the graph of f is concave up for $x > e$.

(C) f is increasing for $x > 1$, and the graph of f is concave down for $x > e$.

(D) f is increasing for $x > 1$, and the graph of f is concave up for $x > e$.

(E) f is increasing for $0 < x < e$, and the graph of f is concave down for $0 < x < e^{3/2}$.

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27. If f is the function given by $f(x) = \int_4^{2x} \sqrt{t^2 - t}\, dt$, then $f'(2) =$

(A) 0 (B) $\frac{7}{2\sqrt{12}}$ (C) $\sqrt{2}$ (D) $\sqrt{12}$ (E) $2\sqrt{12}$

28. If $y = \sin^{-1}(5x)$, then $\frac{dy}{dx} =$

(A) $\frac{1}{1+25x^2}$

(B) $\frac{5}{1+25x^2}$

(C) $\frac{-5}{\sqrt{1-25x^2}}$

(D) $\frac{1}{\sqrt{1-25x^2}}$

(E) $\frac{5}{\sqrt{1-25x^2}}$

END OF PART A OF SECTION I

IF YOU FINISH BEFORE TIME IS CALLED, YOU MAY CHECK YOUR WORK ON PART A ONLY.

DO NOT GO ON TO PART B UNTIL YOU ARE TOLD TO DO SO.